Elsa - A semi-analytical model for estimating CO_2 leakage from aquifers

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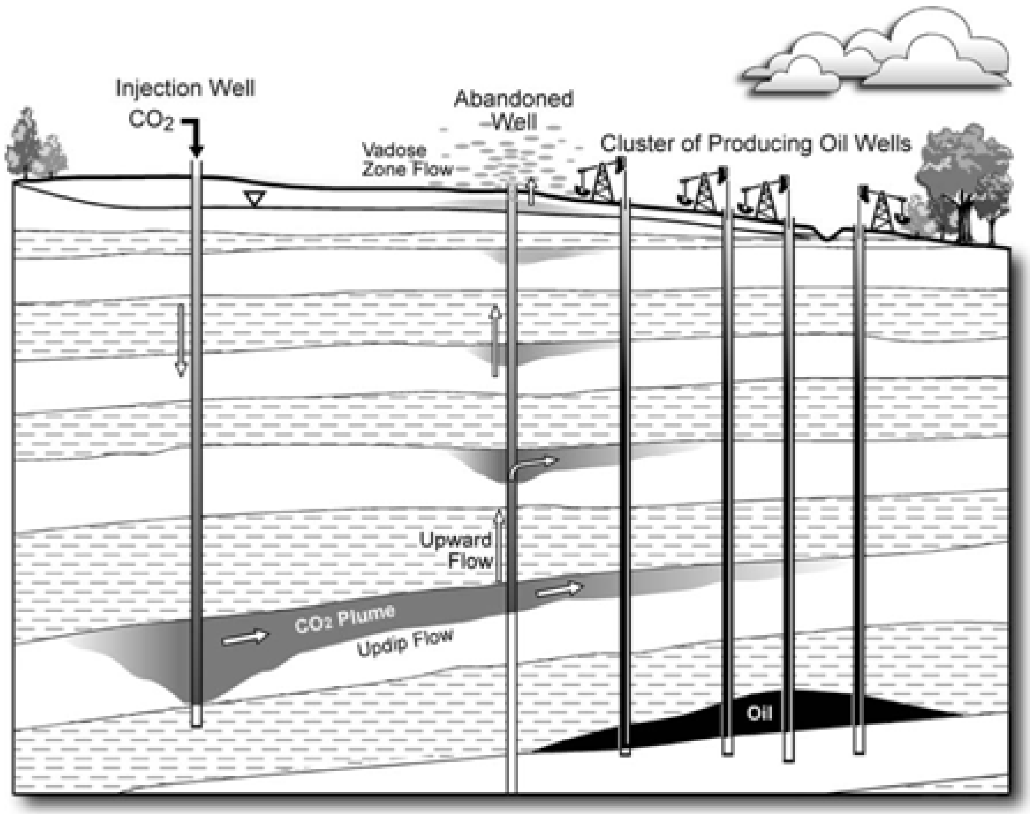
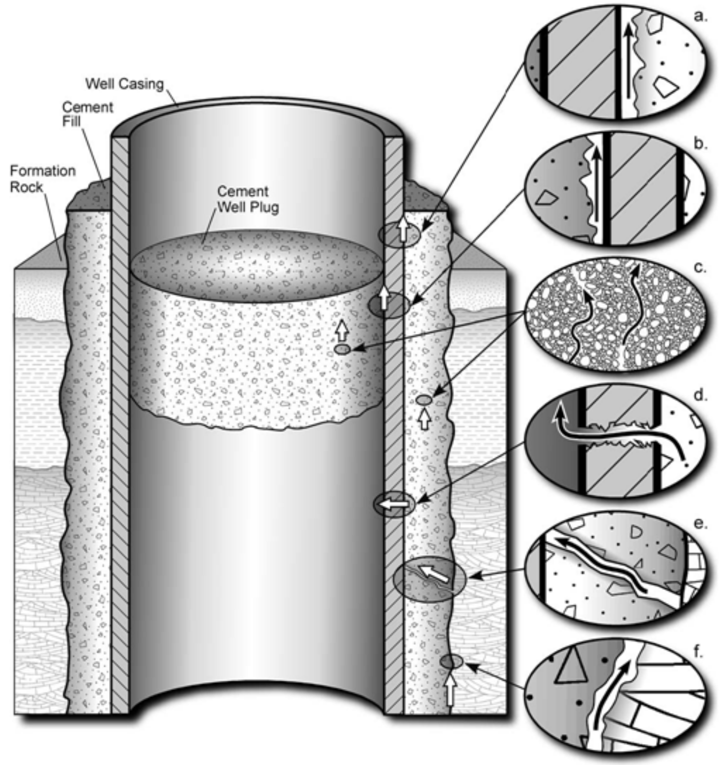
June 1st 2005

Overview

- Motivation for using a semi-analytical approach.
- The main components of the Elsa framework
  - Transport equations.
  - Pressure equations.
  - Saturation equations in the abandoned wells.
- Implementation.
- Application: The Wabamun formations.

A high risk leakage path





The leakage problem

- Initial well properties are usually at best known statistically.
- The degradation of abandoned wells in presence of CO_2 is unknown.
- Well density is known, sometimes also well locations.
- These uncertainties cannot be dealt with in a deterministic manner.
- There is a need for fast and reliable tools for estimating leakage, allowing statistical analysis as e.g. Monte Carlo simulations.

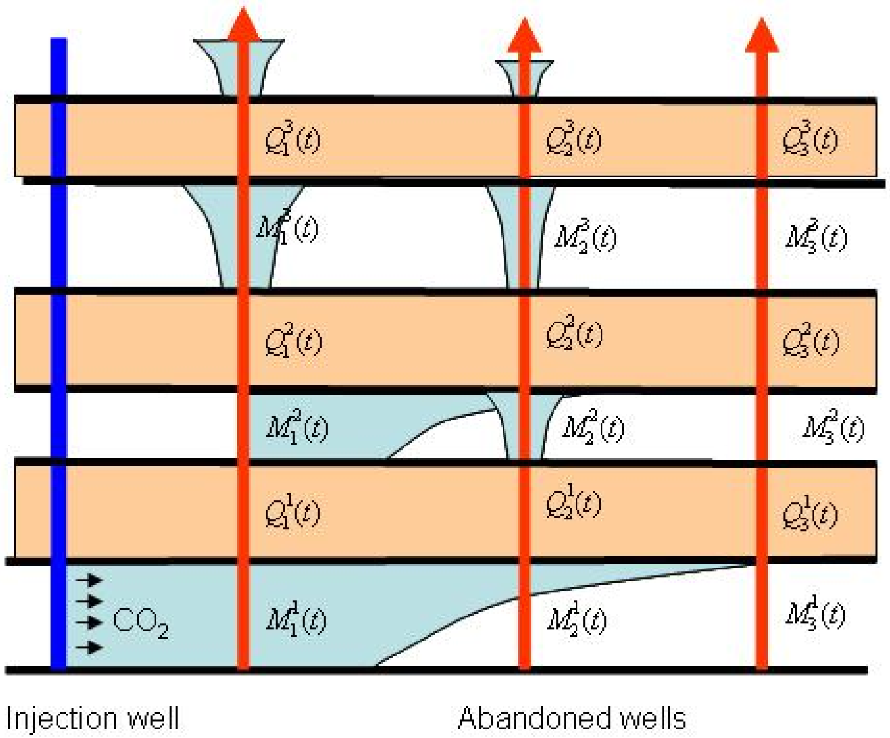
Overall Solution Strategy

- Physical Approximations: Homogeneous horizontal aquifers, inititally at equilibrium, impermeable aquitards, no capillary pressure.

- Mathematical Approximations: Local radial symmetry around wells, vertical equilibrium, infinite aquifers, low order series expansions, etc.

- Numerical Approximations: Solution strategy of the resulting non-linear differential equations.

Discretization in terms of abandoned wells



The state variables and equations are defined and written for each well segment.

State variables of our system

Symbol	Name	Vector size	Dimension
p_w^l	Pressure at bottom of aquifer	$N_w\,N_f$	[M/L/T^2]
$Q_{\alpha,w}^l$	Mass flux of phase α	$2\,N_w\,N_r$	[M/T]
$M_{\alpha,w}^l$	Plume mass of phase α	$2\,N_w\,N_f$	[M]
s_w^l	CO_2 saturation in well	$2\,N_w\,N_r$	[-]

Sub- and superscripts refer to well w and aquifer/aquitard l.

Simplified system of equations

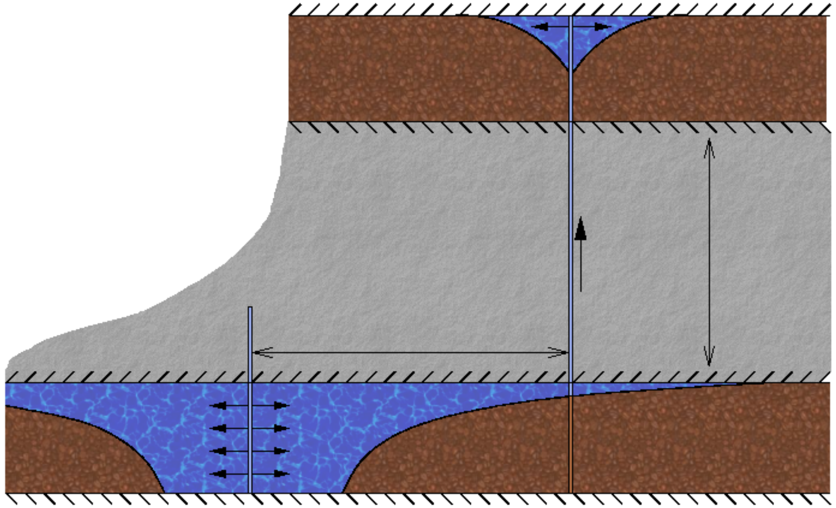
- Mass balance equation: Relates masses to fluxes.
- Darcy equation in wells: Relates fluxes to pressures across aquitards.
- Pressure equation: Relates pressures to fluxes within aquifers.
- Well saturation equation: Relates CO_2 well saturations to the total flowrate for each well.

Major constitutive equation:

- Plume evolution equation: Distributes the masses around wells.

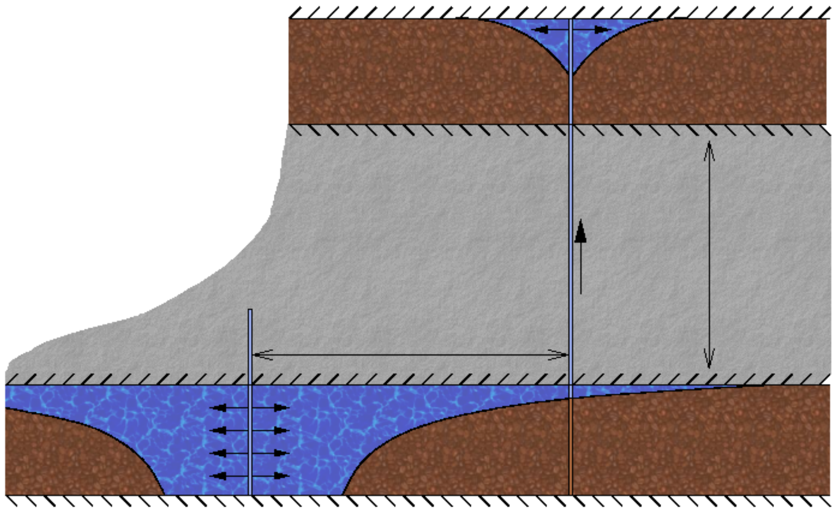
Mass Balance Equation

$$\frac{dM^l_{c,w}}{dt} = \rho_c \sum_{i \in \Omega^l_{\mathrm{in},w}} Q^{l-}_{c,i}(t) - \rho_c \sum_{i \in \Omega^l_{\mathrm{out},w}} Q^{l+}_{c,i}(t)$$



Darcy equation in wells

$$Q^l_\alpha = \pi (r^l)^2 k^l \lambda^l_\alpha (s^l_\alpha) \left(\frac{p_B^{l+} - p_T^{l-}}{H^l} + \rho^l_\alpha g \right).$$



Pressure Equation

Vertically integrated pressure equation:

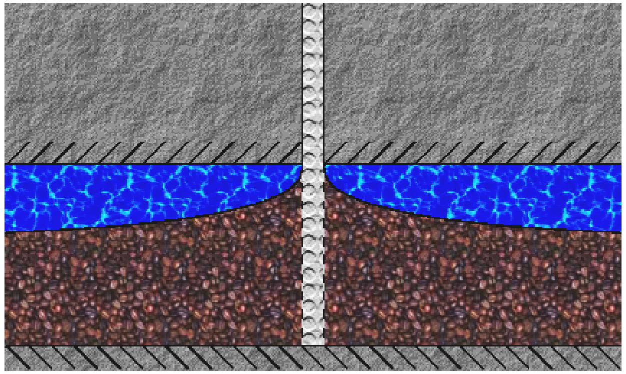
$$c^l_{tot}\frac{\partial \bar{p}^l}{\partial t} - \nabla \cdot (k^l \lambda^l_{\text{eff}} \nabla \bar{p}^l + \rho^l_{\text{eff}} g) = \sum_i \frac{Q_i^{l-} - Q_i^{l+}}{H^l \rho}.$$

Solution in terms of fundamental solutions:

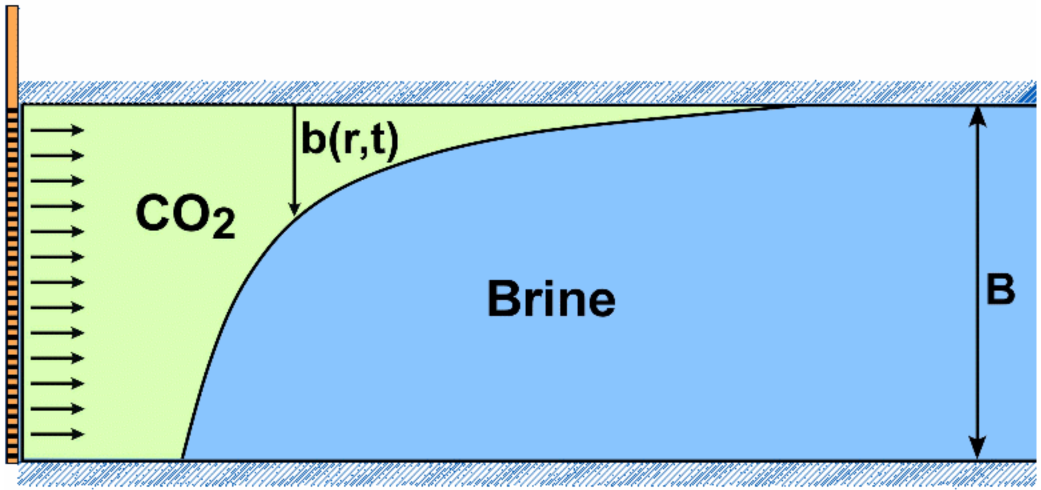
$$\bar{p}^l(\mathbf{x}, t) - \bar{p}^l(t_0) = \sum_{i=0}^{N_{\text{well}}} \left(\frac{d[Q_i^{l-}(t) - Q_i^{l+}(t)]}{dt} *_{t_0}^{t} G(\mathbf{x}, \mathbf{x}_i, h^l(\xi, \tau), t) \right).$$

Well Saturation Equation

$$\lim_{t\to\infty} \frac{Q_b}{Q_b + \frac{\rho_b}{\rho_c} Q_c} = \frac{H^l - h^l}{H^l - h^l(1 - \mu_w/\mu_c)}$$



Plume evolution equations (simplified)



$$b(r,t) = \frac{B}{\lambda_c - \lambda_w}\left(\sqrt{\frac{\lambda_c \lambda_w Q t}{\phi \pi B r^2}} - \lambda_w\right).$$