Elsa - A semi-analytical model for estimating CO$_2$ leakage from aquifers

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Overview

• Motivation for using a semi-analytical approach.

• The main components of the Elsa framework
  – Transport equations.
  – Pressure equations.
  – Saturation equations in the abandoned wells.

• Implementation.

• Application: The Wabamun formations.
A high risk leakage path
The leakage problem

- Initial well properties are usually at best known statistically.
- The degradation of abandoned wells in presence of CO\(_2\) is unknown.
- Well density is known, sometimes also well locations.
- These uncertainties cannot be dealt with in a deterministic manner.
- There is a need for fast and reliable tools for estimating leakage, allowing statistical analysis as e.g. Monte Carlo simulations.
Overall Solution Strategy

- Physical Approximations: Homogeneous horizontal aquifers, initially at equilibrium, impermeable aquitards, no capillary pressure.

- Mathematical Approximations: Local radial symmetry around wells, vertical equilibrium, infinite aquifers, low order series expansions, etc.

- Numerical Approximations: Solution strategy of the resulting non-linear differential equations.
Discretization in terms of abandoned wells

The state variables and equations are defined and written for each well segment.
State variables of our system

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Vector size</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_w^l$</td>
<td>Pressure at bottom of aquifer</td>
<td>$N_w N_f$</td>
<td>[M/L/T$^2$]</td>
</tr>
<tr>
<td>$Q_{\alpha,w}^l$</td>
<td>Mass flux of phase $\alpha$</td>
<td>$2 N_w N_r$</td>
<td>[M/T]</td>
</tr>
<tr>
<td>$M_{\alpha,w}^l$</td>
<td>Plume mass of phase $\alpha$</td>
<td>$2 N_w N_f$</td>
<td>[M]</td>
</tr>
<tr>
<td>$s_w^l$</td>
<td>CO$_2$ saturation in well</td>
<td>$2 N_w N_r$</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Sub- and superscripts refer to well $w$ and aquifer/aquitard $l$. 
Simplified system of equations

- Mass balance equation: Relates masses to fluxes.
- Darcy equation in wells: Relates fluxes to pressures across aquitards.
- Pressure equation: Relates pressures to fluxes within aquifers.
- Well saturation equation: Relates $CO_2$ well saturations to the total flowrate for each well.

Major constitutive equation:

- Plume evolution equation: Distributes the masses around wells.
Mass Balance Equation

\[
\frac{dM_{c,w}^l}{dt} = \rho_c \sum_{i \in \Omega_{in,w}^l} Q_{c,i}^l(t) - \rho_c \sum_{i \in \Omega_{out,w}^l} Q_{c,i}^l(t)
\]
Darcy equation in wells

\[ Q^l_{\alpha} = \pi (r^l)^2 k^l \lambda^l_{\alpha} (s^l_{\alpha}) \left( \frac{p^l_{B} - p^l_{T}}{H^l} + \rho^l_{\alpha} g \right) \]
Pressure Equation

Vertically integrated pressure equation:

\[ c_t^{l} \frac{\partial \bar{p}^{l}}{\partial t} - \nabla \cdot (k^{l} \lambda_{\text{eff}} \nabla \bar{p}^{l} + \rho_{\text{eff}} g) = \sum_{i} \frac{Q_{i}^{l-} - Q_{i}^{l+}}{H^{l} \rho}. \]

Solution in terms of fundamental solutions:

\[ \bar{p}^{l}(x, t) - \bar{p}^{l}(t_0) = \sum_{i=0}^{N_{\text{well}}} \left( \frac{d[Q_{i}^{l-}(t) - Q_{i}^{l+}(t)]}{dt} \right)_{t_0}^{t} \ast G(x, x_{i}, h^{l}(\xi, \tau), t). \]
Well Saturation Equation

\[
\lim_{t \to \infty} \frac{Q_b}{Q_b + \frac{\rho_b}{\rho_c} Q_c} = \frac{H^l - h^l}{H^l - h^l(1 - \mu_w/\mu_c)}
\]
Plume evolution equations (simplified)

\[ b(r, t) = \frac{B}{\lambda_c - \lambda_w} \left( \sqrt{\frac{\lambda_c \lambda_w Q t}{\phi \pi B r^2}} - \lambda_w \right). \]