Carbon geosequestration modelling using semi-analytical solutions

Probabilistic Analysis of Leakage Scenarios
Part 2

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Physical process model

Why is it more efficient than full numerical models?

1. State variables are only defined at well locations
2. No grid, thus no need for grid refinement around wells
3. Equations are relatively fast to evaluate (no infinite series, etc.)

Semi-analytical model may be 100-1000’s times faster than full numerical models, eg, hours vs weeks
Physical process model
The big picture

\[ \frac{dM^l_{\alpha,w}}{dt} = \sum_{i \in \Omega_{\alpha,w}(t)} Q^{l-}_{\alpha,i}(t) - \sum_{i \in \Omega_{\alpha,w}(t)} Q^{l+}_{\alpha,i}(t) \]

\[ Q^l_{\alpha,w}(t) \propto k(S^l_{\alpha,w}) K^l_w \left[ \frac{P^{l-}_{w,T} - P^{l+}_{w,B}}{H^l} - \rho_{\alpha} g \right] \]

\[ p^l_w(t) = p^l_w(t_0) + \sum_{i=1}^{N_w} \sum_{\alpha} Q^l_{\text{net},\alpha,w} G_i(r_{w,j}, \Theta) \]

\[ h^l_w = F_h(\Theta) \]
\[ S^l_{\alpha,w} = F_S(Q^l_{\alpha,w}, \Theta_S) \]

\[ \frac{dM}{dt} = F_M(M, Q) \]
\[ Q = F_Q(p, S) \]
\[ p = F_p(M, Q, h) \]

\[ h = F_h(M, Q) \]
\[ S = F_S(Q, h) \]

Mathematics of the equations
- System of nonlinear differential-algebraic equations (DAEs)
- Initial-value problem (state variables vary in time)
- Large sparse system: \( N_{\text{unknowns}} \propto (N_{\text{wells}} \times N_{\text{layers}}) \)

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Physical process model
Numerical approximation – Scheme A

Numerics of the equations

• Nonlinearity of the time dependence requires time stepping scheme

  \[ M(t^0) \rightarrow M(t^1) \rightarrow M(t^n) \rightarrow M(t^{Nt}) \]

• Nonlinearity of the algebraic equations requires iterative solver

\[
\frac{dM}{dt} = F_M(M, Q) \\
Q = F_Q(p, S) \\
p = F_p(M, Q, h) \\
h = F_h(M, Q) \\
S = F_S(Q, h)
\]

\[
\begin{align*}
\{F_p, F_Q\} &: (M^n, Q^n, p^n) \rightarrow (p^{n+1}, Q^{n+1/2}) \\
\{F_Q, F_S\} &: (p^{n+1}) \rightarrow (Q^{n+1}, S^{n+1}) \\
\{F_M\} &: (M^n, Q^{n+1}) \rightarrow (M^{n+1})
\end{align*}
\]

Explicit time stepping (analogous to IMPES)

Requires the solution of a sparse nonlinear system (“pressure solve”) \(\rightarrow\) Linearise and solve a sparse linear system

Simultaneous solution for pressures at all locations
Physical process model
Numerical approximation – Scheme B

Numerics of the equations
• The equations can be evaluated sequentially (nonlinear Gauss-Seidel)

\[ \frac{dM}{dt} = F_M(M, Q) \]
\[ Q = F_Q(p, S) \]
\[ p = F_p(M, Q, h) \]
\[ h = F_h(M, Q) \]
\[ S = F_S(Q, h) \]

\[ F_M: (M^n, Q^{n+1,i}) \rightarrow (M^{n+1,i+1}) \]
\[ F_h: (M^{n+1,i+1}, Q^{n+1,i}) \rightarrow (h^{n+1,i+1}) \]
\[ F_p: (Q^{n+1,i}, M^{n+1,i+1}, h^{n+1,i+1}) \rightarrow (p^{n+1,i+1}) \]
\[ F_S: (Q^{n+1,i}, h^{n+1,i+1}) \rightarrow (S^{n+1,i+1}) \]
\[ F_Q: (p^{n+1,i+1}, S^{n+1,i+1}) \rightarrow (Q^{n+1,i}) \]

Implicit time stepping (more stable)
Does not requires separate solver for the “pressure solve”
(simpler algorithm)

Multiple iterations \( i \) within a single time step \( n \)
Monte Carlo simulations

• “Known” formation properties and well locations (Edmonton, Alberta, Canada: 7 aquifers, 500 wells)

• **Assumed well permeability distributions**
  * Bimodal Gaussian mix

\[
\log[K] \sim \begin{cases} 
N(\mu_1, \sigma_1^2) & \text{with } p_1 \\
N(\mu_2, \sigma_2^2) & 1-p_1
\end{cases}
\]

\[
\begin{align*}
\mu_1 &= -16; \quad \mu_2 = -20; \quad (\log_{10}[m^2/s]) \\
\sigma_1^2 &= \sigma_2^2 = 2
\end{align*}
\]

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Plume spread – Single realisation

Layer above injection, after 30 yrs

Symbols scaled with mass associated with each well in the selected layer

Figure: iRun=1; iT ime=500; iLayer=1

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Plume evolution – Averaged

Layer above injection, after 30 yrs

Symbols scaled with mass associated with each well in the selected layer

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Leakage statistics

After 30 yrs

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Mass distribution in layers

The elevator effect

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What to do if well properties (even their distributions) are unknown?

• Usually, question is ~ “given current knowledge of system, estimate probability of exceeding the allowable leakage”

• In absence of well statistics, this question is difficult to answer …

• But can analyse ~ “what kind of distributions should the wells (system) satisfy so that leakage risk is X%”

• Will require extensive numerical simulations to identify such relations if they exist

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Preliminary conclusions

• Possible to approximate the plume physics using fast semi-analytical methods
• Open avenues for systematic analysis of leakage probabilities, with view to risk assessment, monitoring networks, etc.
• Work in progress …